Memory Based_JEE Main Online Test_07-01-20_Evening

PHYSICS

1. An electron of mass m and a photon have nearly equal energy then find $\frac{\lambda_e}{\lambda_p} = ?$

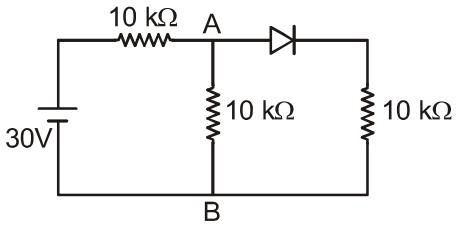
Sol. $\lambda_e = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

$\lambda_p = \frac{hC}{E}$

$\frac{\lambda_e}{\lambda_p} = \frac{E}{C}\frac{1}{\sqrt{2mE}}$

$\frac{\lambda_e}{\lambda_p} = \frac{1}{C}\sqrt{\frac{E}{2m}}$

2. Find out potential difference across AB.

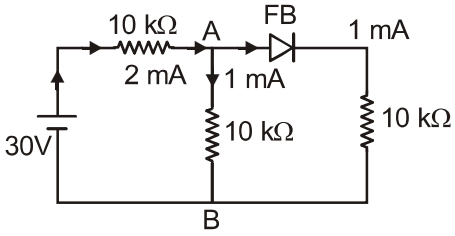


Sol. Diode is forward Bias

$i = \frac{30}{15\times10^3}$ = 2 mA (from battery)

This current divides equally in both resistor of resistance 10 kΩ which in 1 mA

$(V_A - V_B) = \left(10\times10^3 \times \frac{1}{10^3}\right)$ = 10 volt



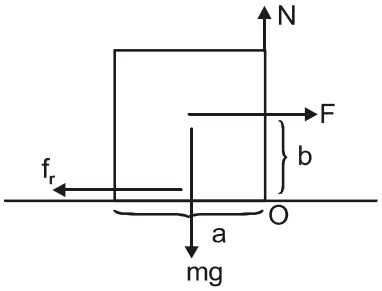
3. In YDSE experiment distance between the slits and screen D = 1.5 m, distance between the slits d = 0.15 mm and wavelength λ = 580 nm of light find out fringe width.

Sol. $\beta = \frac{\lambda D}{d} \Rightarrow \quad \beta = \frac{589\times10^{-9}\times1.5}{0.15\times10^{-3}}$

$\beta = 589 \times 10^{-5}$ m

$\beta = 5.89$ mm

4. A cubical block of side length a is kept of a rough surface with friction coefficient $\mu = 0.4$. A force F is acting at a height b above the surface as shown in the figure then, find ratio of a/b such that block is just about to topple?



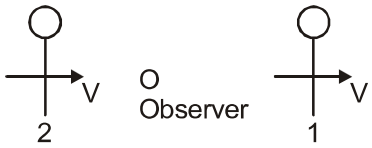
Sol. τ about O; $Fb = mg.\frac{a}{2} \Rightarrow \frac{F}{mg} = \frac{a}{2b}$

$F = \mu mg \Rightarrow \frac{F}{mg} = 0.4$

$\frac{a}{2b} = \frac{4}{10} \Rightarrow \frac{a}{b} = \frac{4}{5}$

5. A stationary observer 'S'. Ore turing bark is coming towards and moving a way from the stationary observer with same velocity 'v'. Observer is hearing 2 beats.
(Frequency of the tuning fork = 1400 Hz) (Velocity of sound = 350 m/s)

Sol.



$F_1 = \left(\frac{V_s}{V_s + V}\right)F_0$

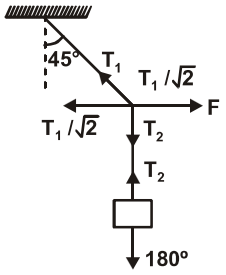
$F_2 = \left(\frac{V_s}{V_s - V}\right) \times F_0$

$f_2 - f_1 = 2$
$V_s f_0 V = V_s^2 - V^2$
$V^2 + 490{,}000\ V - 122500 = 0$
$V = 0.25$ m/s

$V = \frac{1}{4}$ m/s

6. A 10 kg mass is hanging from a 4m long rope attached to a rigid support when a force F is applied at the mid-point of the rope the upper half of the rope forms an angle of 45° from the vertical then find F.

Sol. Here, according to figure,



$T_2 = 100 = \frac{T_1}{\sqrt{2}}$

and $F = \frac{T_1}{\sqrt{2}} = 100$ N

7. Find dimensional formula of energy density = $\frac{1}{2}B^2/\mu_0$

Sol. Energy density = $\frac{1}{2}\frac{B^2}{\mu_0}$

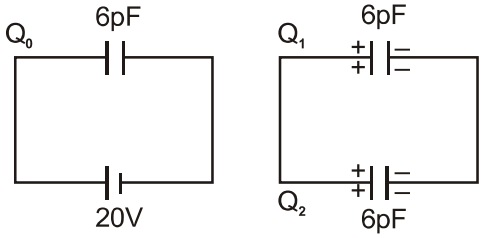
$$\left[\frac{B^2}{2\mu_0}\right]=\left[\frac{E}{V}\right]=\frac{ML^2T^{-2}}{L^3}=ML^{-1}T^{-2}$$

8. A lens ($\mu = 1.5$) is placed in water ($\mu = 1.42$) then find the ratio of focal length in water and air.

Sol.
$$\frac{\frac{1}{f_a}=(\mu-1).\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}{\frac{1}{f_w}=\left(\frac{\mu_L}{\mu_W}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}$$

$$\frac{f_w}{f_a}=\left(\frac{\frac{3}{2}-1}{\frac{1.5}{1.42}-1}\right)=\frac{0.5}{0.056}=8.92$$

9. A 6pF capacitor is charged by a 20V supply. It is then disconnected from the supply and is connected to another uncharged 6pF capacitor. How much electrostatic energy ofthe first capacitor is lost in the form of heat.



Sol. $C_1 = 6pF$ $V_1 = 20V$

$C_2 = 6pF$ $V_2 = 0V$

$$\Delta U = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1-V_2)^2$$

$$\Delta U = \frac{1}{2}\times\frac{6\times10^{-12}\times6\times10^{-12}}{6\times10^{-12}+6\times10^{-12}}(20-0)^2$$

$$\Delta U = \frac{1}{2}\times\frac{6\times10^{-12}}{12\times10^{-12}}\times400$$

$\Delta U = 6 \times 10^{-10}$ J

10. Activity of a radioactive meterial is 700/sec after 30 min it becomes 500/sec then find half life of the material.

Sol. $A = A_0 e^{-\lambda t}$

$$\frac{500}{700} = e^{-\lambda t}$$

$$\lambda = \frac{\log(7/5)}{30}$$

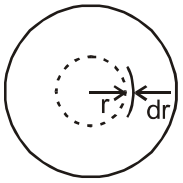
$$T_{1/2} = \frac{30\log(2)}{\log(7/5)} \approx 62 \text{ min}$$

11. A non-uniform disc of radius R has surface mass density $\sigma = A + Br$ where r is distance from centre of disc. Then find moment of inertia of disc.

Sol. $dI = dm \cdot r^2$

$$= \sigma 2\pi r dr \cdot r^2$$

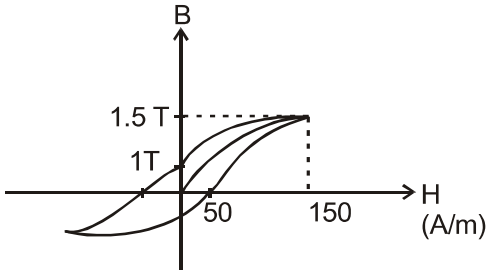
$$= \int_0^R (A + Br) 2\pi r^3 dr$$



$$I = \frac{A2\pi R^4}{4} + \frac{B2\pi R^5}{5}$$

$$I = 2\pi R^4 \left[\frac{A}{4} + \frac{BR}{5}\right]$$

12. For a given hysterisis curve for ferromagnetic material.



Find Coercivity, Retentivity and saturation magnetic field.

Sol. From the given graph:

Coercivity = 50 A/m

Retentivity = 1 T

Saturation M.F. = 1.5 T

13. m gram steam at 100ºC is mixed with 200 gm ice at 0ºC which results in water at 40ºC Then find the m.

Sol. Law of Calorimetry $\Rightarrow$ Heat Lost = Heat gained.

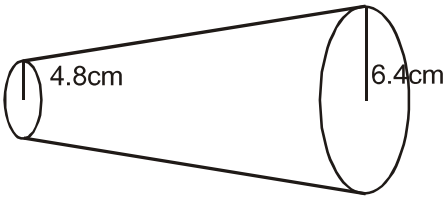
$m \times 540 + m \times 1 \times 60 = 200 \times 80 + 200 \times 1 \times 40$

$\Rightarrow$ $60\ m = 1600 + 800$

$$\Rightarrow \quad m = \frac{2400}{60}$$

$\Rightarrow$ $m = 40$ gm

14. Fluid is flowing through a conical tube shown below, the find the ratio of maximum velocity two the minimum velocity for the flow of the fluid.



Sol. $A_1 V_1 = A_2 V_2$

$$\pi r_1^2 V_1 = \pi r_2^2 V_2$$

$$\frac{V_1}{V_2} = \left(\frac{r_2^2}{r_1^2}\right) = \left(\frac{6.4}{4.8}\right)^2$$

15. The gravity at the North Pole is 10 m/s^2. The weight of the object at North Pole is 196 N. Radius of the earth is 6400 km. Find the weight at the equator.

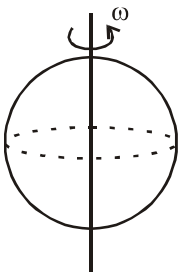
(A) 194.66 (B) 195.32 (C) 194.32 (D) 195

Ans. **(B)**

Sol. $g_{\text{North Pole}} = 10$

The weight of the object at North Pole = 196 N.

Radius of the earth = 6400 km.



at Equator:

$g' = g - \omega^2 R$

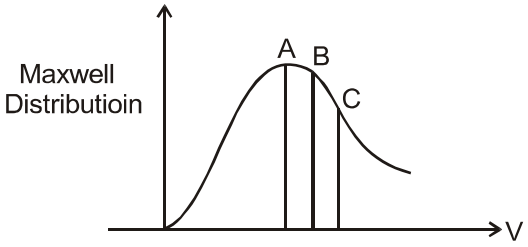
$W' = mg - m\omega^2 R$

$$W' = 196 - 19.6\left(\frac{2\pi}{T}\right)^2 \times 6400 \times 10^3$$

$$W' = 196 - 19.6\,\frac{4\times\pi^2}{(24\times 3600)^2} \times 6400 \times 10^3$$

$W' = 195.32$

16. In the following Maxwell Distribution curve which point represent RMS, Average and most probable speed of the paticle.

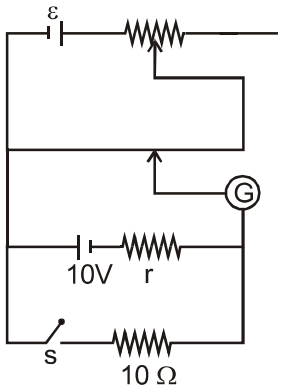


Sol. A $\rightarrow$ Most Probable

B $\rightarrow$ Average

C $\rightarrow$ RMS

17. In the given potentiometer arrangement null point ocurs at 560 cm when switch S is open & at 500 cm when S is closed then find value of internal resistance r.



Sol. Let ϕ is potential gradient in potentiometer wire.

here, when switch s is open

$$\varepsilon = 560\,\phi \qquad ...(i)$$

when s is closed

$$\varepsilon - ir = 500\,\phi \qquad ...(ii)$$

where $i = \dfrac{\varepsilon}{10+r}$

Then, $\varepsilon - \dfrac{\varepsilon r}{10+r} = 500\phi$ ...(iii)

Dividing equation (i) & (iii)

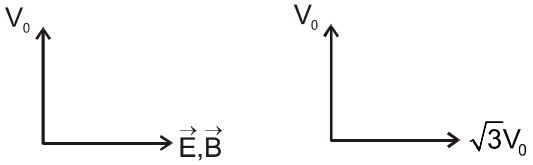
$$\Rightarrow \quad \frac{\varepsilon}{\frac{\varepsilon(10+r-r)}{10+r}} = \frac{560\phi}{500\phi}$$

$$\Rightarrow \quad \frac{10+r}{10} = \frac{28}{25}$$

$$\therefore \quad r = \frac{6}{5}$$

18. $E = E_0\hat{i}$, $B = B_0\hat{i}$, $V = V_0\hat{j}$ Find the time, when speed becomes $2V_0$.

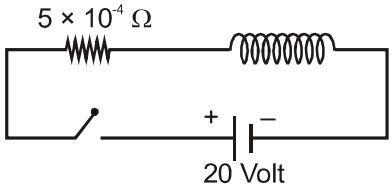
Sol.



$$\sqrt{3}V_0 = \frac{qE}{m}t$$

$$\frac{\sqrt{3}mV_0}{qE} = t$$

19. Find the ratio of current at t = ∞ sec and t = 40 sec. ($e^2 = 7.73$)

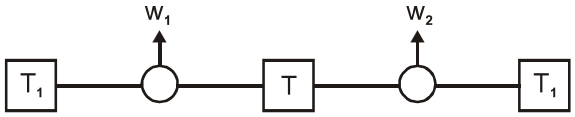


Sol.

$$\frac{i_\infty}{i_{(40\,sec)}} = \frac{V/R}{\frac{V}{R}\left(1-e^{\frac{-RT}{L}}\right)}$$

$$\frac{i_\infty}{i_{(40\,sec)}} = \frac{1}{\left(1-e^{\frac{-RT}{L}}\right)} = \frac{7.73}{(7.73-1)} \approx 1.15$$

20. Two reservior with hat one at T_1 and cold one of 2nd reservoir at T_2 such that sink of Ist is source of second if work done by both the engine are equal ($w_1 = w_2$) then find temperature T.



Sol. $w_1 = w_2$

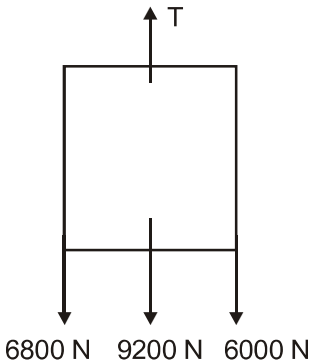
$T_1 - T = T - T_2$

$T_1 + T_2 = 2T$

$$T = \left(\frac{T_1 + T_2}{2}\right)$$

21. A lift of mass 920 kg arries 10 persons each of mass 68 kg with velocity 3m/s and friction of 6000 N act opposite to motion then find power delivered by lift ?

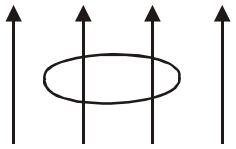
Sol.



$\therefore$ T = 22000 N

P = T × V = 22000 × 3 = 66000 Watts.

22. A disc rotating in a uniform magnetic field of time period = 10 sec. then find maximum and minimum E.M.F



Sol. $\phi = BA\cos(\omega t)$

$|\varepsilon| = \left|\frac{d\phi}{dt}\right| = BA\sin(\omega t)$

Maximum E.M.F

$\omega t = \frac{\pi}{2}$

$\frac{2\pi}{T}t = \frac{\pi}{2}$

$t = \frac{T}{4} = 2.5$ sec

Minimum E.M.F

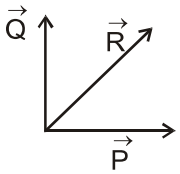
$\omega t = \pi$

$\frac{2\pi}{T}t = \pi$

$t = \frac{T}{2} = 5$ sec

23. $\vec{P}$ & $\vec{Q}$ are 2 vectors and a third vector $\vec{R} = \vec{P} + \vec{Q}$ such that $|\vec{R}| = |\vec{P}|$. then angle between $2\vec{P} + \vec{Q}$ and $\vec{Q}$ is = ?

Sol.



$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$0 = Q(Q + 2P\cos\theta) = 0$

$Q = 0$ (Not possible)

$Q + 2p\cos\theta = 0 \Rightarrow |Q + 2P\cos\theta = 0|$

$\left(2\vec{P} + \vec{Q}\right)\ \vec{Q}$

$2\vec{P}\cdot\vec{Q} + Q^2$

$2PQ\cos\theta + Q^2$

$= Q(2P\cos\theta + Q) = 0$

Angle between $\left(2\vec{P} + \vec{Q}\right)$ & $\vec{Q}$ is 90°

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CHEMISTRY

1. Among the given pair, the element with greater value of electron gain enthalpy (S, Se), (Cl, F), (Na, Li)

Ans. **S,Cl, Li**

2. For cyanide CN^-, the correct option is

Ans. **Bond order = 3 , Diamagnetic**

3. In [Ma_2b_2] complex, number of optical isomers for tetrahedral and square planar geometry is :

Ans. **optical isomers sp^3 = 0**

dsp^2 = 0

4. If cobalt (III), is approached by strong field ligand.

Ans. **Correct state of statement is to be selected**

5. Cl_2 + NaOH (hot & conc.) $\longrightarrow$ X+ side product

Cl_2 + NaOH(dry) $\longrightarrow$ Y

Product X & Y respectively

Ans. **$NaClO_3$, $Ca(OCl)_2$**

6. If melting point of metal is very low in comparision to impurities associated with it, then which methods is used for refining :

Ans. **Liquation**

7. Which of the following statements are **correct**

(I) Decomposition of hydrogen peroxide produces O_2.

(II) 2-ethyl anthaquinol is used in preparation of H_2O_2.

(III) H_2O_2 is used in preparation of sodium metaperborate.

(IV) Oxygen is released on thermal decompositon of $KClO_3$, $Pb(NO_3)_2$, $NaNO_3$.

Ans. **All the above satements are correct.**

8. Which reaction is redox in nature

(1) Formation of ozone.

(2) Reaction of dinitrogen and dioxygen to form nitric oxide

(3) Acid–Base reaction was given.

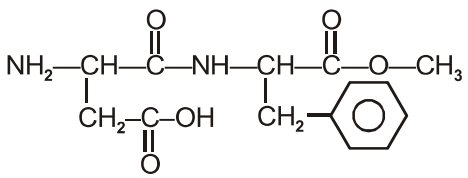
(4) - - - - - - - - - - - - - -

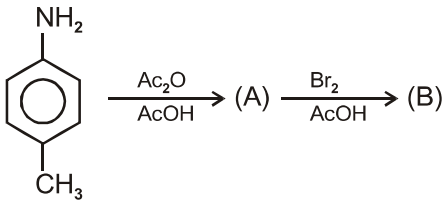
Ans. **Reaction of dinitrogen and dioxygen to form nitric oxide is redox reaction.**

9. Number of sp^2 carbans in Aspartame

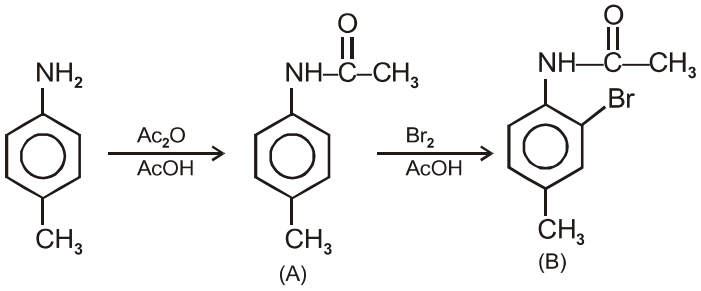
Ans. **9**

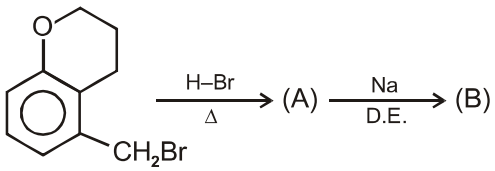
Sol.

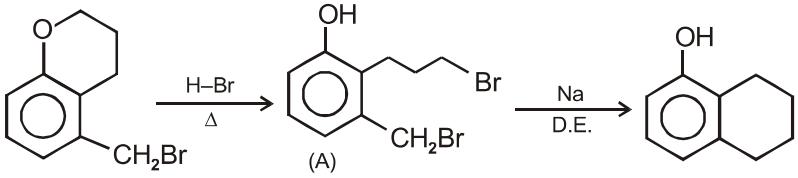


10. 

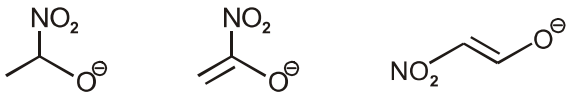
Find (B)

Ans. 

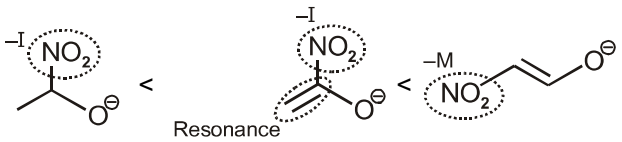
11. 

Ans. 

12. Find correct order of stability of following anions :

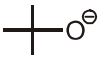


Sol. Stability of carbanion ∝ –M, –I, –H



13. $CH_3CH_2CH_2CH_2–Br$ → Substitution (K_S) / Elimination (K_E) ; $\mu = \frac{K_S}{K_E}$

$CH_3CH_2–O^-$ ..... Z_A

 Z_B

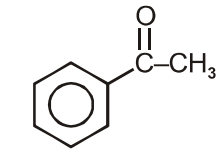
Identify correct option if given compound is react with Z_A and Z_B:

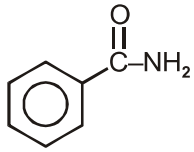
(1) $\mu_A > \mu_B$, $K_{EA} > K_{EB}$ (2) $\mu_A < \mu_B$, $K_{EB} > K_{EA}$ (3) $\mu_A > \mu_B$, $K_{EA} < K_{EB}$ (4) $\mu_A < \mu_B$, $K_{EA} > K_{EB}$

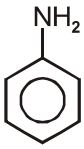
Ans. **(3)**

$\mu_A > \mu_B$, $K_{EA} < K_{EB}$

14. A column is packed with aniline, Acetophenon and Benzamide. Silica gel is taken as stationary phase with eluent hexane and ethyl acetate in 20 : 80 ratio. Identify the order from bottom to top.


Acetophenone


Benzamide


Aniline

Ans. Acetophenone > Benzamide > Aniline (Bottom to Top)

15. Correct statement:

(1) Glucose reaction with conc. HNO_3 to give gluconic Acid

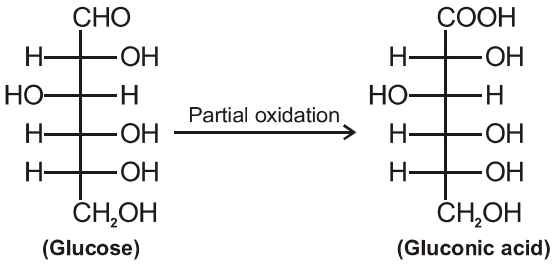
(2) Gluconic Acid is a dicarboxylic acid

(3) Gluconic Acid is formed by partial oxidation of glucose

(4) Gluconic acid pyranose form is hemiacetal or ketal

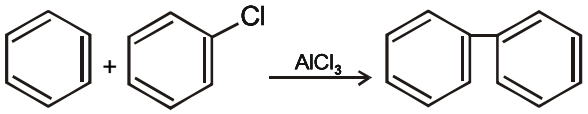
Ans. **(3)**

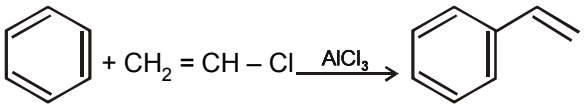
Gluconic Acid is formed by partial oxidation of glucose.

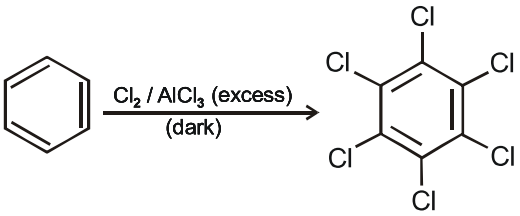


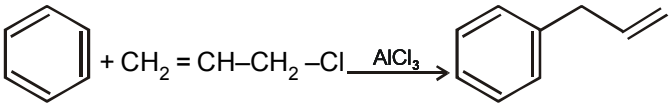
(Glucose) **(Gluconic acid)**

16. Identify the correct reaction:

(a) 

(b) 

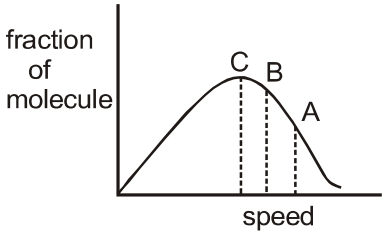
(c) 

(d) 

(1) a, b, c (2) b, c, d (*3) c, d (4) a, d, c

Ans. **(3)**

17. Find U_{rms} U_{mps} U_{avg} from the following curve.



Identify U_{rms} , U_{mps} , U_{avg} = ?

Sol. U_{mps} = C U_{avg} = B U_{rms} = A

18. 3 gm of CH_3COOH is mixed with 0.1 M, 200 ml HCl solution. This solution is diluted up to 500 ml, 20 ml of the solution is reacted with 5 M , 0.5 ml NaOH solution. Find the pH of the final solution.

Sol. mmole of CH_3COOH = 50 in 500 mL solution

mmole of HCl = 20 in 500 mL solution

In 20 mL solution mmole of CH_3COOH = 2

mmole of NaOH = 2.5

$$CH_3COOH + NaOH \longrightarrow CH_3COONa + H_2O$$

2 1.7

0.3 0 1.7

$$pH = 4.74 + \log \frac{1.7}{0.3} = 5.49$$

19. Enthalpy of combustion of C(s, graphite), $H_2(g)$ and ethane $(C_2H_6)(g)$ are given. Find the enthalpy of formation of $C_2H_6(g)$.

Sol. $2C\ (s, graph.) + 3H_2(g) \longrightarrow C_2H_6(g)$

$\Delta H_f = 2\Delta H_C(C, s, graph.) + 3\Delta H_C(H_2, g) - \Delta H_C(C_2H_6, g)$

20. 3 g of urea reacts with NaOH then NH_3 is released. How much volume of HCl solution is required neutralized the released NH_3.

Sol. $NH_2CONH_2 + NaOH \longrightarrow NaOCN + NH_3 + H_2O$

0.05 mole 0.05 mole

Number of mili mole of HCl required to neutralised produced NH_3 = 50 mmole = molarity × Volume (in ml)

21. Question from chemical kinetics based on reversible reaction to write rate law of backward reaction :

22. Question from surface chemistry to calculate Flocculation value for As_2S_3 sol.

23. Find out incorrect statement :

(1) $\Lambda_m^0(H_2O) = \Lambda_m^0(NaOH) + \Lambda_m^0(HCl) - \Lambda_m^0(NaCl)$

(2) $\Lambda_m^0(NaBr) + \Lambda_m^0(NaCl) = \Lambda_m^0(NaCl) + \Lambda_m^0(KBr)$

(3) $\Lambda_m^0(NaCl) - \Lambda_m^0(NaBr) = \Lambda_m^0(KCl) - \Lambda_m^0(KBr)$

(4) $\Lambda_m^0(NaCl) + \Lambda_m^0(KBr) = \Lambda_m^0(NaBr) + \Lambda_m^0(KCl)$

Ans. **(2)**

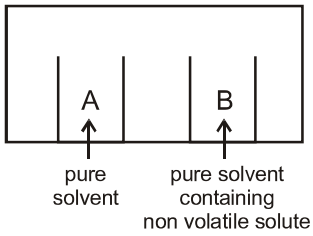
24. Which is redox reaction among the following :

(1) reaction between N_2 and O_2 at 2000° C (2) reaction between $[Co(NH_3)_6]Cl_3$ and $AgNO_3$

(3) reaction between H_2SO_4 and NaOH (4) formation of ozone in presence of sun light

Ans. **(1)**

25.



Affter some time, choose correct option :

(1) Volume of A and B remains same (2) Volume of A decreases and B same

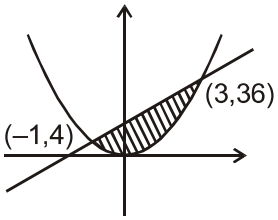
(3) Volume of A same and B decreases (4) Volume of A and B both decreases

Ans. **(4)**

Memory Based_JEE Main Online Test_07-01-20_Evening

MATHEMATICS

1. Consider the region bounded by $4x^2 \le y \le 8x + 12$
Find the area bounded = ?



Hence, area = $\int_{-1}^{3} (8x+12) - (4x^2)\,dx$

$= \frac{148}{3}$

2. $3 + 4 + 8 + 9 + 13 + 14 + \ldots\ldots\ldots$(40 terms) = 102m
then m = ?

Sol. $7 + 17 + 27 + \ldots\ldots$ (20 term)

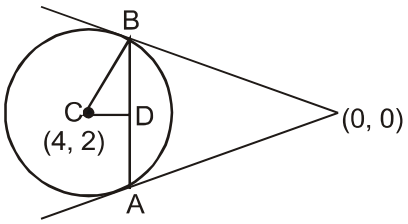
$= \frac{20}{2}[(2 \times 7) + 19 \times 10]$

$= 10[14 + 190] \quad \Rightarrow \quad m = 20$

3. Tangents are drawn from (0, 0) to $x^2 + y^2 - 8x + 4y + 16 = 0$ which touches the circle at A and B Find $(AB)^2$

Ans. **12.8**

Sol. $BC = \sqrt{16+4-16} = 2$



equations of AB
$x.0 + y.0 - 4(x + 0) + 2(y + 0) + 16 = 0$
$-4x + 2y + 16 = 0$
$-2x + y + 8 = 0$

$CD \text{ is } = \left|\frac{2\times4-2-8}{\sqrt{4+1}}\right| = \left|\frac{-8-2+8}{\sqrt{4+1}}\right| = \frac{2}{\sqrt{5}}$

$BD = \sqrt{(CB)^2 - (CD)^2} = \sqrt{4 - \frac{4}{5}} = \sqrt{\frac{16}{5}}$

$= \sqrt{8 - \frac{4}{5}} = \sqrt{\frac{24}{5}}\ \sqrt{\frac{36}{5}} = \frac{4}{\sqrt{5}}$

$AB = 2 \times BD = 8\sqrt{5}$

$AB^2 = \frac{64}{5} = 12.8$

4. There are five machines, prob. of failing of any one of machine on a particular day is $\frac{1}{4}$. The probability that at most two machines are broken is $\left(\frac{3}{4}\right)^3$ k find k

Sol. $q = \frac{1}{4}$

$\Rightarrow P = \frac{3}{4}$

Given ${}^5C_0\, p^5 + {}^5C_1 qp^4 + {}^5C_2 q^2p^3 = \frac{27}{64}$ k

$k = \frac{34}{16} = \frac{17}{8}$

5. If tangent on $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ is $3x + 4y = 12\sqrt{2}$ find distance between foci.

Ans. $2\sqrt{7}$

Sol. $\therefore$ Tangent is $y = -\frac{3x}{4} = 3\sqrt{2}$

$m = -\frac{3}{4}$, $C = 3\sqrt{2}$

$c^2 = a^2m^2 + b^2$

$18 = \frac{9a^2}{16} + 9$

$9 = \frac{9a^2}{16}$

$a = 4$

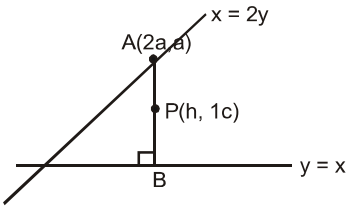
$9 = 16\,(1 - e^2)$

$e = \frac{\sqrt{7}}{4}$

Distance between foci = $2ae = 2\sqrt{7}$

6. Find locus of mid point of perpendicular drown from x = 2y to y = x

Sol.



Equation of AB : $x + y = 3a$

B is P.O.I $\begin{Bmatrix} y = x \\ x + y = 3a \end{Bmatrix} B\left(\frac{3a}{2}, \frac{3a}{2}\right)$

$h = 2a + \frac{3a}{2}$ $\qquad$ $K = a + \frac{3a}{2}$

$\frac{h}{k} = \frac{7}{5}$

$y = \frac{5}{7}x$

7. If $f(x) = x^3 - 4x^2 + 8x + 12$
find 'C' by using L.M.V.T if $x \in [0, 1]$

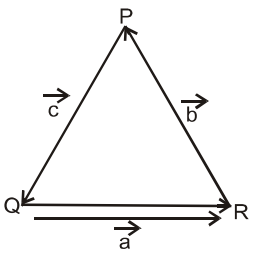
Sol. $f'(C) = 3C^2 - 8C + 8 = \frac{f(1) - f(0)}{1 - 0}$

$3C^2 - 8C + 8 = 5$
$3C^2 - 8C + 3 = 0$

$C = \frac{4 - \sqrt{7}}{3}$

8. Let $\vec{a}, \vec{b}, \vec{c}$ are unit vectors satisfying $\vec{a} + \vec{b} + \vec{c} = \vec{0}, \cdot \vec{d} = \vec{a} \times \vec{b} + \vec{b} + \vec{c} + \vec{c} \times \vec{a}$ and $\lambda = \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ then $\vec{d}$ and λ.

Sol. ΔPQR is equilateral Δ.



$\vec{d} = 3\vec{a} \times \vec{b}$ and

$\lambda = 3(\vec{a}.\vec{b}) = 3\cos 120° = -\frac{3}{2}$

9. Find coefficient of x^7 in S where S is
$S = (1 + x)^{10} + x(1 + x)^9 + x^2(1 + x)^8 + \ldots\ldots + x^{10}$
(1) 120 (*) 330 (3) 240 (4) 420

Ans. **(2)**

Sol. Coefficient is $= {}^{10}C_7 + {}^9C_6 + \ldots\ldots + {}^3C_3$
$= {}^3C_3 + {}^4C_3 + {}^5C_3 + \ldots\ldots + {}^{10}C_3$
$= {}^4C_4 + {}^4C_3 + {}^5C_3 + \ldots\ldots + {}^{10}C_3$
$= {}^{11}C_4$ using using ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

10. Let $k \in I$; $r \in I$ and $6\ {}^{35}C_r = (k^2 - 3)\ {}^{36}C_{r+1}$. Find number of all ordered pair (r, k)

Ans. **4**

Sol. $6 \cdot {}^{35}C_r = (k^2 - 3)\ \dfrac{36}{r+1} \cdot {}^{35}C_r$

$$k^2 - 3 = \frac{r+1}{6}$$

$r = 6k^2 - 19$

as $r \in [0, 35]$

$k = \pm 2, \pm 3$

11. If the roots of $x^2 - x - 1 = 0$ are α and β ; and $P(K) = \alpha^k + \beta^k$; then which of the following is incorrect .

(1) $P(5) = P(2).P(3)$

(2) $P(1) + P(2) + P(3) + P(4) + P(5) = 26$

(3) $P(5) = 11$

(4) $P(5) = P(4) + P(3)$

Sol. $P(1) = 1$

$P(2) = (\alpha + \beta)^2 - 2\alpha\beta = 3$

$P(3) = \alpha^3 + \beta^3$

$= \alpha(\alpha + 1) + \beta(\beta + 1)$

$= 3 + 1 = 4$

$P(4) = \alpha^4 + \beta^4 = (\alpha^2 + \beta^2) - 2\alpha^2\beta^2$

$= 9 - 52 = 7$

$P(5) = \alpha^5 + \beta^5 = \alpha(\alpha^4) + \beta(\beta^4)$

$= \alpha(\alpha + 1)^2 + \beta(\beta + 1)^2$

$= \alpha(\alpha^2 + 2\alpha + 1) + \beta(1^2 + 2\beta + 1)$

$= 4 + 2(3) + 1$

$= 11$

12. Let $A = [a_{ij}]_{3\times3}$ be square matrix of 3×3.

Let $B = [b_{ij}]_{3\times3}$ be a matrix such that

$b_{ij} = 3^{i+j-2}\, a_{ji}$

if $|B| = 81$ find $|A|$.

Sol. $$|B| = \begin{vmatrix} a_{11} & 3a_{21} & 3^2a_{31} \\ 3a_{12} & 3^2a_{22} & 3^3a_{32} \\ 3^2a_{13} & 3^3a_{23} & 3^4a_{33} \end{vmatrix}$$

$|B| = 3^3\, 3^3\, |A|$

$|B| = 81 = 3^6|A|$

$$|A| = \frac{1}{9}$$

13. Let f(x) be a polynomial of degree 5 if $\lim_{x\to0}\left(2+\frac{f(x)}{x^3}\right) = 4$. Let x = ±1 are critical points, then which of the following is incorrect.

(1) f(1) is maxima and f(–1) is minima. (*2) f(1) is minima and f(1) is maxima.

(3) f(1) – 4f(–1) = 4 (4) f(x) is odd function.

Ans. **(2)**

Sol. $\lim_{x\to0}\frac{f(x)}{x^3} = 2$

$\therefore \quad f'(x) = kx^2(x^2-1)$

$$f(x) = k\left(\frac{x^5}{3}-\frac{x^3}{3}\right)+C$$

$f(0) = 0 \Rightarrow \quad C = 0$

$$\lim_{x\to0}\frac{f(x)}{x^3} = 2$$

$$\Rightarrow \quad k\left(-\frac{1}{3}\right) = 2$$

$k = -6$

$$f(x) = 2x^3 - \frac{6}{5}x^5$$

14. If $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$

and $y\left(\frac{1}{2}\right) = -\frac{1}{4}$, find $\frac{dy}{dx}$ at $x = \frac{1}{2}$

Sol. Let $x = \sin\alpha$

$y = \sin\beta$

$\therefore \quad \sin(\alpha+\beta) = k$

$\alpha + \beta = \sin^{-1}k$

or $\quad \sin^{-1}x + \sin^{-1}y = \sin^{-1}k$

$$\Rightarrow \quad \frac{1}{\sqrt{1-x^2}}+\frac{1}{\sqrt{1-y^2}}\,y' = 0$$

$$\Rightarrow \quad y' = -\sqrt{\frac{1-y^2}{1-x^2}}$$

$$\therefore \quad (y')_{x=\frac{1}{2}} = -\sqrt{\frac{1-\frac{1}{16}}{1-\frac{1}{4}}} = -\sqrt{\frac{15}{12}} = -\sqrt{\frac{5}{4}} = -\frac{\sqrt{5}}{2}$$

15. Let $a_1\ a_2$ . . .. a_n be a G.P. If $a_1 < 0$ and $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $a_1 + a_2 + ... + a_9 = 4\lambda$ Find λ ?

Sol. $a + ar = 4$

$ar^2 + ar^3 = 16$

Divide : $r^2 = 4$

$r = 2$ or -2

$a = -4$ and $r = -2$

$$\text{Sum} = a\left(\frac{r^9 - 1}{r - 1}\right) = -4\left(\frac{(-2)^9 - 1}{-3}\right)$$

$$= -\frac{4}{3}(2^9 + 1) = 4\lambda$$

$$\lambda = -\frac{513}{3} = -171$$

16. If $4\alpha \int_{-1}^{2} e^{-\alpha|x|}dx = 5$

find α

Sol. $$4\alpha\left(\int_{-1}^{0} e^{\alpha x}dx + \int_{0}^{2} e^{-\alpha x}dx\right) = 5$$

$$4\left(e^{\alpha x}\Big|_{-1}^{0} - e^{-\alpha x}\Big|_{0}^{2}\right) = 5$$

$$4\left[\left(1 - e^{-\alpha}\right) - \left(e^{-2\alpha} - 1\right)\right] = 5$$

$4e^{-2\alpha} + 4e^{-\alpha} - 3 = 0$

$e^{-\alpha} = -\frac{3}{2}$ or $\frac{1}{2}$

$\alpha = \ln 2$

17. Solution of differential equation $(y^2 - x)\dfrac{dy}{dx} = 1$ cuts the x axis at A. Find A . (where y(0) = 1/2)

Sol. $\dfrac{dx}{dy} + x = y^2$

$x.e^y = \int y^2 e^y dy$

$xe^y = e^y.y^2 - 2(ye^y) + 2e^y + c$

$x = y^2 - 2y + 2 + ce^{-y}$

$0 = \dfrac{1}{4} - 1 + 2 + c\ e^{-\frac{1}{2}}$ (use $y(0) = \dfrac{1}{2}$)

$C = -\dfrac{5}{4}\sqrt{e}$

$A : y = 0 \Rightarrow x = 2 + c$

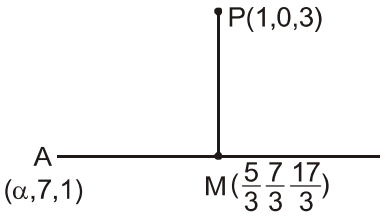
$x = 2 - \dfrac{5}{4}\sqrt{e}$

Subjective :

18. The foot of perpendicular from point P(1, 0, 3) on a line is $\left(\dfrac{5}{3}, \dfrac{7}{3}, \dfrac{17}{3}\right)$. If line passes through $(\alpha, 7, 1)$, find value of α.

Ans. **4**

Sol.



$AM \perp PM$

$\left(\alpha - \dfrac{5}{3}\right)\left(1 - \dfrac{5}{3}\right) + \left(7 - \dfrac{7}{3}\right)\left(0 - \dfrac{7}{3}\right) + \left(1 - \dfrac{17}{3}\right)\left(3 - \dfrac{17}{3}\right) = 0$

$\Rightarrow \quad \alpha = 4$

19. $\begin{cases} \frac{1}{x}\log\frac{(1+3x)}{1-2x} & x \neq 0 \\ K & x = 0 \end{cases}$

is continuous at x = 0, Find K -

Ans. **5**

Sol. $f(0) = K = \lim\limits_{x\to 0} \dfrac{\log\left(\dfrac{1+3x}{1-2x}\right)}{x}$

$= \lim\limits_{x\to 0} \dfrac{\log(1+3x) - \log(1-2x)}{x}, \dfrac{0}{0}$

by using L' Hospital

$= \lim\limits_{x\to 0} \dfrac{\dfrac{1\times 3}{1+3x} - \dfrac{1(-2)}{1-2x}}{1}$

$= \dfrac{3}{1+0} + \dfrac{2}{1-0} = 5$

20. If mean and variance of the data 3, 7, 9, 12, 13, 20, x, y are 10 and 25 respectively ; find x.y.

Sol. x + y = 16 (As $\bar{x} = 10$)

& $x^2 + y^2 = 148$ (As $\sigma^2 = 25$)

$\therefore$ xy = 54

21. X = {x |x $\in$ N ; $\cap \in$ [1, 50]

A = {n |n = 2λ ; $\lambda \in$ N}

B = {m |m = 7λ ; $\lambda \in$ N}

Find minimum number of elements in subset of X such that it contains all elements of both A & B.

Sol. A = {2, 4, 6,. . .....50}

B = {7, 14,. . .....49}

A $\cap$ B = {14, 28, 42}

Hence minimum number of elements = 3.

22. x + y + z = 6

x + 2y + 3z = 10

3x + 2y + λz = μ has more than two solution .Find $\mu - \lambda^2$.

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & \lambda \end{vmatrix} = 0$

and $D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 3 & 2 & \mu \end{vmatrix} = 0$

$\mu = 14$

$\mu - \lambda^2 = 14 - 1 = 13$