Memory Based_JEE Main Online Test_08-01-20_Evening

Physics

1. Frequency of electromagnetic wave is 15 GHz at a point P magnetic field $\vec{B} = (5\times10^{-8})\,\hat{j}$ wave propagate in z direction then find $\vec{E}$ at point P.

Sol. $\hat{E}\times\hat{B} = \hat{C}$

$\hat{B} = \hat{j}$, $\hat{C} = \hat{k}$

$\hat{E} = \hat{i}$

$\frac{E}{B} = C$

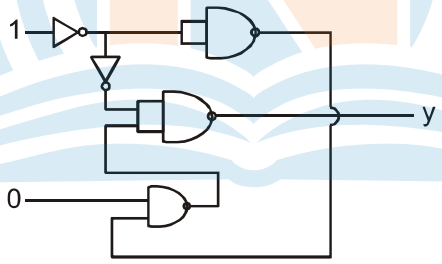
$E = CB$

$E = (3 \times 10^8) \times (5 \times 10^{-8})$

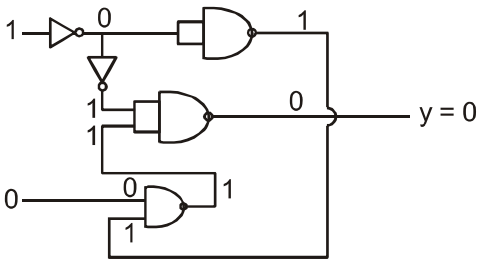
$E = 15$ V/m

$\vec{E} = 15\,\hat{i}$

2. In the given figure, find the output 'y'.

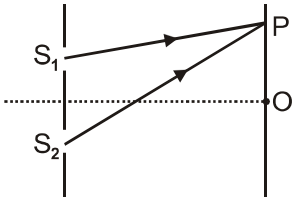


Sol.



There fore y = 0.

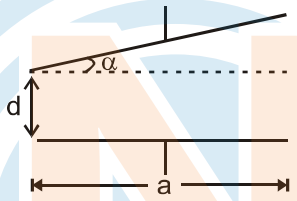
3. In YDSE setup path difference at point P is $\lambda/8$. Then find ratio of intensity at P and at point O.



Sol. $I_P = I_0 \cos^2\left(\frac{\Delta\varphi}{2}\right) = I_0 \cos^2\left[\frac{1}{2}\frac{2\pi}{\lambda}\Delta x\right] = I_0 \cos^2\left[\frac{\pi}{8}\right]$

$$\frac{I_P}{I_0} = \cos^2\left(\frac{\pi}{8}\right) = \left[\frac{1+\cos\pi/4}{2}\right] = \left(\frac{1+\frac{1}{\sqrt{2}}}{2}\right) = \left(\frac{\sqrt{2}+1}{2\sqrt{2}}\right)$$

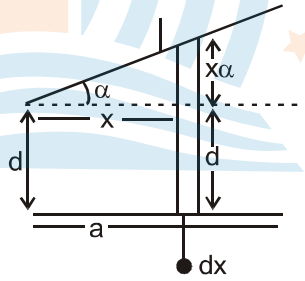
4. Two square conducting plates of area a^2 place a shown in figure find out the capacitance of the capacitor. (α is very small)



Sol. $\int dc = \int_0^a \frac{\varepsilon_0 (a\,dx)}{(d+x\alpha)}$

$$C = \frac{\varepsilon_0 a}{\alpha}\left[\ell n(d+x\alpha)\right]_0^a$$

$$C = \frac{\varepsilon_0 a}{\alpha}\ell n\left(\frac{a\alpha+d}{d}\right)$$



$$C = \frac{\varepsilon_0 a}{\alpha}\ell n\left(1+\frac{a\alpha}{d}\right) \qquad \left(\ell n(1+x) \approx x - \frac{x^2}{2}\right)$$

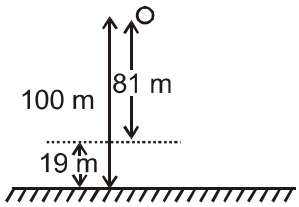
$$C = \frac{\varepsilon_0 a}{\alpha}\left(\left(\frac{a\alpha}{d}\right) - \frac{\left(\frac{a\alpha}{d}\right)^2}{2}\right)$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 a}{\alpha}\left(\frac{a\alpha}{d}\right)\left(1-\frac{a\alpha}{2d}\right)$$

$$C = \frac{\varepsilon_0 a^2}{d}\left(1-\frac{a\alpha}{2d}\right)$$

5. A particle is released from rest at height 100 m above the surface of a planet. In last $\left(\frac{1}{2}\text{sec}\right)$ this particle travels 19 m distance then find gravitational acceleration near the planet surface.

Sol.



Let total time t,

$$100 = \frac{1}{2}at^2 \qquad ...(i)$$

$$81 = \frac{1}{2}\times a\times\left(t-\frac{1}{2}\right)^2 \qquad ...(ii)$$

On dividing (i) and (ii)

$$\frac{10}{9} = \frac{t}{t-\frac{1}{2}} \Rightarrow t = 5$$

$$100 = \frac{1}{2}\times a\times 25 \Rightarrow a = 8 m/s^2$$

6. Length of simple pendulum is measured ℓ = 25.0 cm and time of 40 oscillations of pendulum is measured 50 sec by using a stopwatch of resolution 1 sec. Find out the percentage error in estimation of g.

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$g = \frac{4\pi^2\ell}{T^2}$$

t = nT

$$\frac{\Delta t}{t} = \frac{\Delta T}{T}$$

$$\frac{1}{50} = \frac{\Delta T}{T}$$

$$\frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + \frac{2\Delta T}{T} = \frac{0.1}{25.0} + 2\times\frac{1}{50}$$

$$\% \text{ error} = \frac{\Delta g}{g}\times 100 = 4.4\%$$

7. A steel wire of length ℓ and mass m. The velocity of wave is V, when the tension is T. The velocity becomes $\frac{V}{2}$ when the tension is T'. Find the value of T'.

Sol. $V = \sqrt{\frac{T}{\mu}}$ .....(i)

$\frac{V}{2} = \sqrt{\frac{T'}{\mu}}$ .....(ii)

Taking the ratio of (i) and (ii)

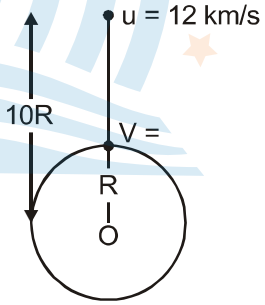
$2 = \sqrt{\frac{T}{T'}}$

$4 = \frac{T}{T'}$

$T' = \frac{T}{4}$

8. A particle at a height 10R from the centre of Earth and its velocity is 12 km/sec towards centre of Earth. Find the velocity of particle on Earth surface. The escape velocity of a particle from the Earth surface is 11.2 km/sec.

Sol. $\left(\frac{1}{2}mu^2\right) + \left(\frac{-GMm}{10R}\right) = \frac{1}{2}mV^2 - \frac{GMm}{R}$



$\frac{1}{2}mV^2 = \frac{1}{2}mu^2 + \frac{GMm}{R}\left(1 - \frac{1}{10}\right)$

$\frac{V^2}{2} = \frac{u^2}{2} + \frac{9}{10}\frac{Gm}{R}$

$V = \sqrt{u^2 + \frac{18}{10}\frac{Gm}{R}}$

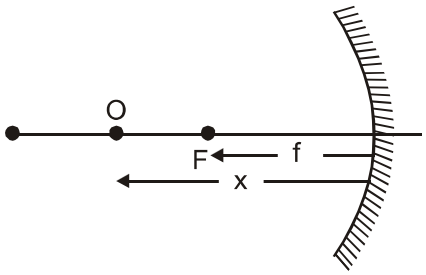
$V = \sqrt{144 + \frac{9}{10}(11.2)^2}$

$V = \sqrt{256.89}$

$V \approx 16$ km/s

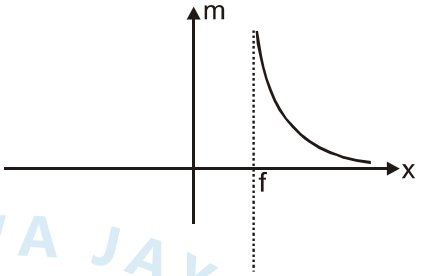
9. A point object starts from focus of concave mirror of focal length f away from the mirror draw the plot between magnification and the distance x where x is distance from the mirror.

Sol.

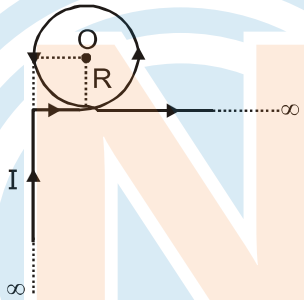


$$m = -\frac{V}{u} = \frac{f}{F-u}$$

$$|m| = \left|\frac{F}{f-x}\right|$$



10. Current in the wire is I than find value of intensity of magnetic filed at point O.



Sol. $\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$

$$\vec{B}_1 = \frac{\mu_0 i}{2R}(\hat{k})$$

$$\vec{B}_2 = \frac{\mu_0 i}{4\pi R}(\sin 90° - \sin 45°)(-\hat{k})$$

$$\vec{B}_3 = \frac{\mu_0 i}{4\pi R}(\sin 90° + \sin 45°)(\hat{k})$$

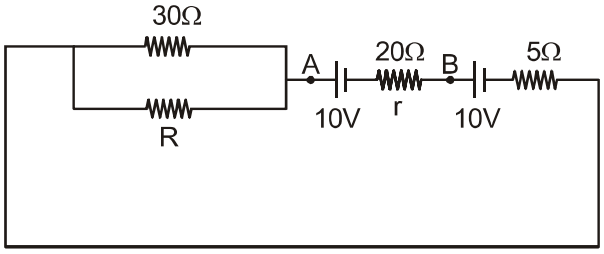
$$B = \frac{\mu_0 i}{2R} + \frac{\mu_0 i}{4\pi R}\left(1 + \frac{1}{\sqrt{2}}\right) - \frac{\mu_0 i}{4\pi R}\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$B = \frac{\mu_0 i}{2R} + \frac{\mu_0 i}{4\pi R}\sqrt{2}$$

$$B = \frac{\mu_0 i}{2R}\left(1 + \frac{\sqrt{2}}{2\pi}\right)$$

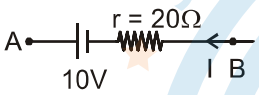
$$B = \frac{\mu_0 i}{2\pi R}\left(\pi + \frac{1}{\sqrt{2}}\right)$$

11. Find the value of R so, That potential drop across (AB) is zero.



Sol. $\Rightarrow \quad \left(\frac{30R}{30+R}+25\right) I = 20$

$$I = \left(\frac{20}{\frac{30R}{30+R}+25}\right)$$



$$10 = \left(\frac{20}{\frac{30R}{30+R}+25}\right)20$$

$$I = \frac{40}{\frac{30R}{30+R}+25}$$

$\Rightarrow \quad \frac{30R+25(30+R)}{(30+R)} = 40$

$\Rightarrow$ 6R + 5 (30 + R) = 8 (30 + R)

$\Rightarrow$ 6R + 150 + 5R = 240 + 8 R

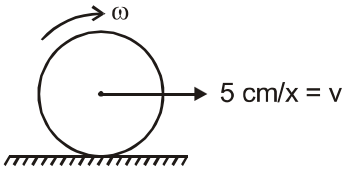
3R = 240 – 150

3R = 90

R = 30 Ω

12. A solid sphere of mass 500g is rolling on a sufficiently rough surface with a velocity of 5cm / sec then find the kinetic energy of the rolling sphere ?

Sol.



$V = \omega R$

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}.\frac{2}{5}mv^2$$

$$KE = \frac{7}{10}mv^2 = \frac{7}{10}\times\frac{1}{2}\times(5\times10^{-2})^2$$

$$= \frac{7}{20}\times25\times10^{-4}$$

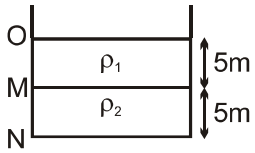
$$KE = \frac{35}{4}\times10^{-4}$$

K.E. = 8.75×10^{-4} J

13. A container containing mixture of two gases Helium and oxygen, n and 2n moles respectively. Find γ mix.

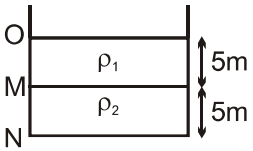
Sol. $$\gamma_{mix} = \frac{C_{p\,mix}}{C_{v\,mix}} = \frac{n_1C_{p_1} + n_2C_{p_2}}{n_1C_{v_1} + n_2C_{v_2}} = \frac{n\times\frac{5}{2}R + 2n\times\frac{7}{2}R}{n\times\frac{3}{2}R + 2n\times\frac{5}{2}R} = \frac{\frac{5}{2}+7}{\frac{3}{2}+5};$$ $$Y_{mix} = \frac{19}{13}$$

14. A beaker of height 10m contains two different liquids of densities of ρ_1 and ρ_2. The lengths of both the liquids are same. Also $\rho_2 = 2\rho_1$. Find the ratio of force applied on OM and MN.



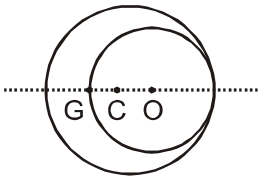
Sol. $$F_{OM} = P_{avg.}\ A_{eff} = \rho_1g\times\frac{5}{2}\times(k\times5)$$

$$F_{MN} = \left(\rho_1g\times5 + \rho_1g\times\frac{5}{2}\right)\times(k\times5)$$



$$\frac{F_{OM}}{F_{MN}} = \frac{\rho_1g\times\frac{5}{2}}{\rho_1g5 + \rho_2g\times\frac{5}{2}} = \frac{\rho_1\times\frac{5}{2}}{\rho_1\times5 + \rho_1\times5} = \frac{5}{2\times10} = \left(\frac{1}{4}\right)$$

15. A disc of radius R having a hole of radius 1m such that its centre is at O and the radius of geometrical centre of the disc is at C , has its centre of mass at point G then which of the following is correct ?

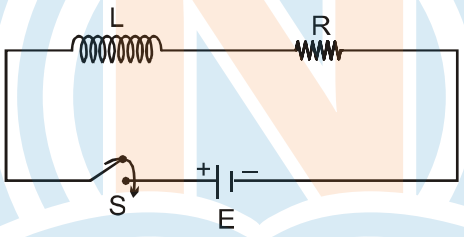


Sol. Taking G at the origin.

$$O = \frac{6\pi R^2(2-R) - 6\pi(1)}{6\pi R^2 - 6\pi(1)}$$

$O = R^2(2-R) - 1$
$O = 2R^2 - R^3 - 1$
$O = R^3 - 2R^2 + 1$
$O = R^2(R-1) + (1-R)(1+R)$
$O = (R-1)(R^2 - (1+R))$
$O = (R-1)(R^2 - R - 1)$

16. Find total charge suplied by the battery from t = 0 to t = τ



Sol. $i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$

$$dQ = \int_0^{L/R} \frac{\varepsilon}{R}\left(1 - e^{-\frac{Rt}{L}}\right)dt$$

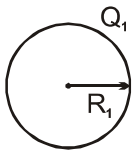
$$Q = \frac{\varepsilon}{R}\left[t + \frac{L}{R}e^{-\frac{Rt}{L}}\right]_0^{L/R}$$

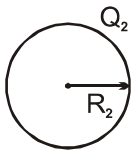
$$Q = \frac{\varepsilon}{R}\left[\frac{L}{R} + \frac{L}{R}.e^{-1} - \frac{L}{R}\right]$$

$$Q = \frac{\varepsilon L}{R^2 e}$$

17. Two spheres having charge Q_1 and Q_2 and radii R_1 and R_2 respectively have electric field at their surfaces in the ratio of $R_1 : R_2$, then find ratio of potentials at their surface ?

Sol.





$$E_1 = \frac{KQ_1}{R_1^2} \qquad E_2 = \frac{KQ_2}{R_2^2}$$

$$V_1 = \frac{KQ_1}{R_1} \qquad V_2 = \frac{KQ_2}{R_2}$$

$$E_1 = \frac{V_1}{R_1} \qquad E_2 = \frac{V_2}{R_2}$$

$$\frac{E_1}{E_2} = \frac{V_1}{V_2} \times \left(\frac{R_2}{R_1}\right) \Rightarrow \frac{V_1}{V_2} = \left(\frac{R_1}{R_2}\right)^2$$

18. A particle of mass m and charge e having velocity $\vec{v}_0 = v_0\hat{i} + v_0\hat{j}$ has debroglie wavelength λ_0, is released in an electric field $\vec{E} = E_0\hat{k}$ then find debroglie wavelength at time t ?

Sol. Here, accleration $(\vec{a}) = \frac{eE_0}{m}\hat{k}$

$$\therefore \quad v_{(z_t)} = \frac{eE_0 t}{m} \qquad \Rightarrow \lambda_0 = \frac{h}{mv} = \frac{h}{m\sqrt{2}\,v_0}$$

$$\vec{v}_{(t)} = v_0\hat{i} + v_0\hat{j} + \frac{eE_0 t\hat{k}}{m}$$

$$\therefore \quad \lambda_{(t)} = \frac{h}{m\,|\vec{v}_{(t)}|} = \frac{h}{m\sqrt{2\,v_0^2 + \frac{e^2E_0^2t^2}{m^2}}}$$

$$= \frac{h}{\sqrt{2}\,mv_0\sqrt{1+\frac{e^2E_0^2t^2}{2m^2v_0^2}}} = \frac{\lambda_0}{\sqrt{1+\frac{e^2E_0^2t^2}{2m^2v_0^2}}}$$

19. There are three container C_1 , C_2 , C_3 having liquid of same specific heat at temperature T_1 , T_2 and T_3 respectively. Following table shows the mixtures taken in 1st , 2nd and 3rd steps, find the T_{mix} at the 4th steps as table suggest.

		Container			
		C_1	C_2	C_3	Tmix(ºC)
Ist step	Volume	1 Litre	2 Litre	-	60
2st step	Volume	-	1 Litre	2 Litre	30
3rd step	Volume	2 Litre	-	-	60
4th step	Volume	1 Litre	1 Litre	1 Litre	θ

Sol. 1st $1 \times S (T_1 - 60) + 2 \times S (T_2 - 60) = 0$

$T_1 + 2 T_2 = 180$ ...(1)

2nd $1 \times S (T_1 - 30) + 2 \times S (T_3 - 30) = 0$

$T_2 + 2 T_3 = 90$ ...(2)

3rd $2 \times S (T_1 - 60) + 1 \times S (T_3 - 60) = 0$

$2T_1 + T_3 = 180$ ...(3)

(1), (2) and (3) $T_1 + T_2 + T_3 = \frac{450}{3} = 150$

4th $1 \times S (T_1 - T) + 1 \times S (T_2 - T) + 1 \times 3 (T_3 - T) = 0$

$T = \frac{T_1 + T_2 + T_3}{3} = 50^oC$

20. Refrigerator is an apparatus which takes heat from a cold body. Work done an working substance is 10J and coefficient of performance = $\frac{1}{10}$. Then find heat extracted from cold reservoir.

Sol. $\text{C.O.P.} = \frac{\text{Heat extracted from cold Reservoir}}{\text{Work done on working substance}}$

$\frac{1}{10} = \frac{\text{Heat extracted}}{10J}$

Heat extracted = 1J

Memory Based_JEE Main Online Test_08-01-20_Morning

Chemistry

1. Atomic radii order of F, O, C, Cl, Br.

Sol. Increasing order : F < O < C< Cl < Br

2. Decreasing order of Bond Energy for C— F, C — Cl, C — Br, C — I.

Sol. C— F > C — Cl > C — Br > C — I

3. $\text{Metal} + N_2 \longrightarrow M_3N_2 \xrightarrow{H_2O} M(OH)_2 + NH_3\uparrow \xrightarrow{CuSO_4} [Cu(NH_3)_4]SO_4$ (deep blue solution)

Sol. $Mg + N_2 \longrightarrow Mg_3N_2$

4. The reaction that does not happen in blast Furnace

(i) $CaO + SiO_2 \longrightarrow CaSiO_3$

(ii) $FeO + SiO_2 \longrightarrow Fe\,SiO_3$

(iii) $FeO \longrightarrow Fe + \frac{1}{2} O_2$

(iv) $Fe_2O_3 + CO \longrightarrow Fe_3O_4 + CO_2$

Sol. (ii) $FeO + SiO_2 \longrightarrow Fe\,SiO_3$

(iii) $FeO \longrightarrow Fe + \frac{1}{2} O_2$

5. Consider and complexes :

$Ni(CO)_4$, $[Ni(CN)_4]^{2-}$, $[Ni(H_2O)_6]^{2+}$, $[PdCl_2(PPh_3)_2]$

The correct order for magnetic moment :

Sol. $[Ni(H_2O)_6]^{2+} > [PdCl_2(PPh_3)_2] > [Ni(CN)_4]^{2-} \approx Ni(CO)_4$

6. When white phosphorus reacts with sodium hydroxide phosphine gas is released along with compound X. X on reaction with HCl form an acid with the basicity.

Sol. $P_4 + NaOH \longrightarrow PH_3 + NaH_2PO_2$ (Sodium hypo phosphite)

$\downarrow$ HCl

H_3PO_2

Basicity of H_3PO_2 = 1

7. ML_5 acquires two geometry

(i) Square pyramidal (ii) trigonal bipyramidal

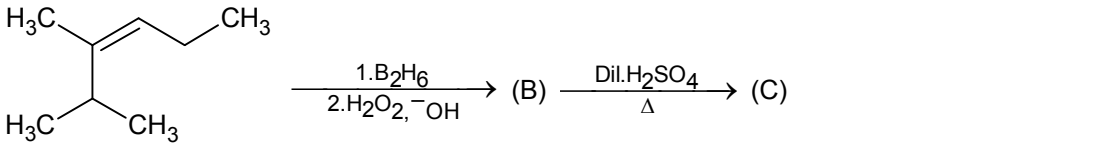
Total number of 90° = x

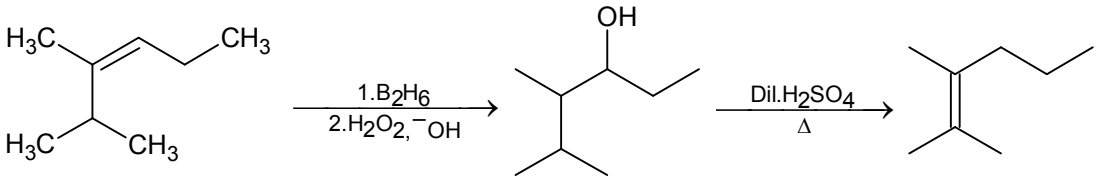
Total number of 120° = y

Total number of 180° = z

Sol. 20

	(i) Square pyramidal		(ii) trigonal bipyramidal		
Total number of 90° = x	8	+	6	=	14
Total number of 120° = y	0	+	3	=	3
Total number of 180° = z	2	+	1	=	3

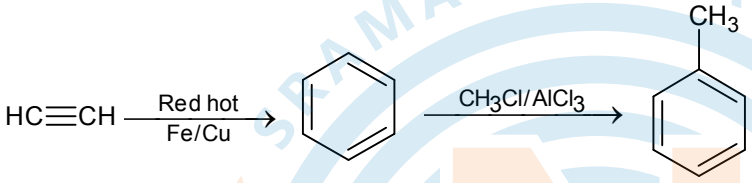
8. 

Sol. 

9. (A) (Smallest Alkyne) $\xrightarrow[Fe/Cu]{\text{Red hot}}$ (B) $\xrightarrow{CH_3Cl/AlCl_3}$ (C)

Maximum number of atoms in a single plane.

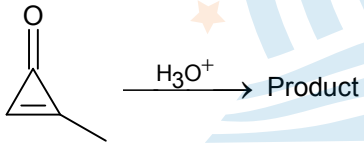
Ans. **13**

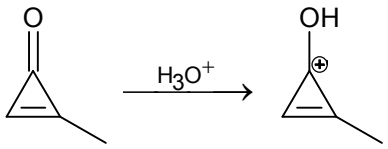
Sol. 

Total 13 atoms are present in one plane.

10. Name the monomers of maltose ?

Sol. α-D glucose + α-D glucose

11. 

Sol. 

Product is aromatic so it is highly stable.

12. Which reaction is involved in formation of Bakelite ?

Sol. Electrophilic substitution and dehydration.

13. Which of the following can't be separated by Khjeldhal method.

(1) CH_3CH_2CN

(2) Urea

(3) Aniline

(4) Nitrobenzene

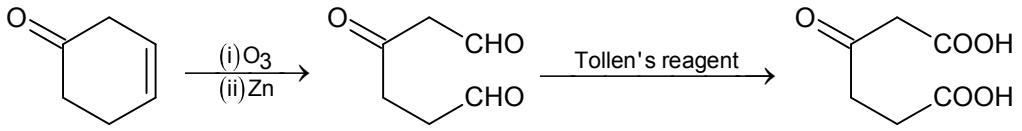
Ans. (4)

Sol. Khjeldhal method is not applicable with nitro, diazo and ring nitrogen.

14. (A) $\xrightarrow[(ii)Zn]{(i)O_3}$ (B) $\xrightarrow{\text{Tollen's reagent}}$ (C) 3-oxo hexane dicarboxylic acid

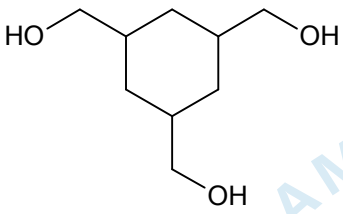
Find the correct structure of (A).

Sol.

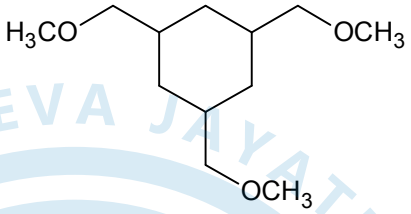


15. A compound with molecular formula $C_9H_{18}O_3$ with double bond equivalue of 1 have two isomers A and B. Boiling point of A > B. Identify the correct structure of A and B

Sol. (A)



(B)



(A) can undergo intermolecular hydrogen bonding so it will have higher boiling point then ether (B).

16. Radius of first Bohr orbit of H atom is a_o Å than calculate radius of second Bohr orbit of Li^{+2} ion

Ans. $\frac{4}{3}a_o$

17. Consider cell representation

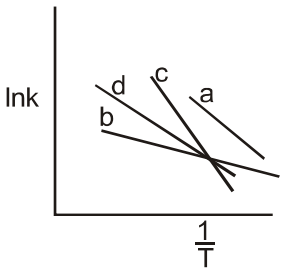
$Sn_{(s)}|Sn^{+2}||Pb^{+2}|Pb_{(s)}$

Given $E^o_{Sn^{+2}/Sn} = -0.14V$ and $E^o_{Pb^{+2}/Pb} = -0.13V$

At equilibrium state determine value of $\frac{[Sn^{+2}]}{[Pb^{+2}]}$

Ans. $10^{1/3} = 2.17$

18.



Compare activation energy of each reaction a, b, c, d.

Sol. $E_a(c) > E_a(a) > E_a(d) > E_a(b)$

19. Assertion - On increasing temperature, pH of water increases .

Reason - Dissociation of water in H^+ and OH^- is an exothermic process.

Ans. Both Assertion and Reason are incorrect.

20. Which of the following show both Schottky and Frenkel defect.

(i) ZnS (ii) CsCl (iii) NaBr (iv) AgBr

Ans. AgBr

21. $NaClO_3 + Fe \longrightarrow NaCl + FeO + O_2$

Calculate mass of $NaClO_3$ required to produce 492 L O_2(g) at 1atm and 300 K which completely consumed by human being per day (R = 0.082 L-atm K^{-1} mol^{-1})

Ans. 2130 g

Memory Based_JEE Main Online Test_08-01-20_Morning

MATHEMATICS

1. How many 4 letters word can be formed using letters of the word EXAMINATION.

Sol. A – 2
E – 1
I – 2
O – 1
X – 1
M – 1
N – 2
T – 1

Case (i) 2 same, 2 same = $^3C_2 \times \frac{4!}{2!2!} = 18$

(ii) 2 same, 2 different, $= {}^3C_1 \times {}^7C_2 \times \frac{4!}{2!}$ = 3 × 21 × 12 = 756

(iii) All different = $^8C_4$ × 4! = 70 × 24 = 1680

Total words = 2454

2. Probability of occurace of exactly one of A or B is $\frac{2}{5}$. Probability of occurance of atleast one of A or B is $\frac{1}{2}$.

Find the probability of there simultaneous occurance.

Sol. Given $P(A) + P(B) - 2P(A \cap B) = \frac{2}{5}$ .....(1)

& $P(A) + P(B) - P(A \cap B) = \frac{1}{2}$ .....(2)

Solve (1) and (2)

$P(A \cap B) = \frac{1}{10}$

3. Given : $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7}$ & $\frac{\sqrt{1-\cos 2\beta}}{\sqrt{2}} = \frac{3}{5}$, find $\tan(\alpha + 2\beta)$.

Sol. $\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha} = \frac{1}{7}$; $\tan\alpha = \frac{1}{7}$

Also $\frac{\sqrt{2}\sin\beta}{\sqrt{2}} = \frac{3}{5}$

$\sin\beta = \frac{3}{5}$; $\tan\beta = \frac{3}{4}$

$$\tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2.\frac{3}{4}}{1-\frac{9}{16}} = \frac{24}{7} ;$$

$$\tan(\alpha+2\beta) = \frac{\tan\alpha + \tan 2\beta}{1-\tan\alpha.\tan 2\beta} = \frac{\frac{1}{7}+\frac{24}{7}}{1-\frac{1}{7}.\frac{24}{7}} = \frac{25\times 7}{25}$$

4. If vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ & $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ satisfies $\vec{b} \times \vec{a} = \vec{b} \times \vec{c}$, $\vec{a}.\vec{c} = 0$, then find $\vec{b}.\vec{c}$

Sol. $\vec{b} \times (\vec{a} - \vec{c}) = 0$

$\Rightarrow \vec{a} - \vec{c} = \lambda\vec{b}$

$\vec{a} = \lambda\vec{b} + \vec{c}$

Dot with $\vec{a} \Rightarrow |\vec{a}|^2 = \lambda\vec{b}.\vec{a} + 0$

$6 = \lambda(4)$

$\lambda = \frac{3}{2}$

Dot with $\vec{b} \Rightarrow \vec{a}.\vec{b} = \lambda|\vec{b}|^2 + \vec{b}.\vec{c}$

$\vec{b}.\vec{c} = \vec{a}.\vec{b} - \lambda|\vec{b}|^2$

$= 4 - \frac{3}{2}(3) = -\frac{1}{2}$

5. If $A = \begin{bmatrix} 2 & 2 \\ 9 & 4 \end{bmatrix}$, then $10A^{-1}$ will satisfy which of the following

(1) A + 6I (2) A – 6I (3) 6I – A (4) ?

Ans. **(2)**

Sol. $A = \begin{bmatrix} 2 & 2 \\ 9 & 4 \end{bmatrix}$

$|A| = -10$

$tr(A) = 6$

$A^2 - 6A - 10I = 0$

$\Rightarrow \quad A^2A^{-1} - 6AA^{-1} - 10I \cdot A^{-1} = 0$

$10A^{-1} = A - 6I$

6. The mean of 20 observations is 10 & variance is 4. One of reading was misread as 9 instead of 11. Find the actual variance.

Sol. Mean = 10 $\rightarrow$ $\Sigma i_i = 200$

$\therefore$ correct $\Sigma x_i = 200 - 9 + 11 = 202$

Also variance = $\frac{\sum x_i^2}{20} - (\bar{x})^2 = 4$

$\Rightarrow \Sigma x_i^2 = (4 + 100) \times 20 = 2080$

$\therefore$ correct $\Sigma x_i^2 = 2080 - 9^2 + 11^2 = 2080 + 40 = 2120$

$\therefore$ correct variance = $\frac{\sum x_i^2}{20} - (\bar{x})^2$

$= \frac{2120}{20} - \left(\frac{202}{20}\right)^2$

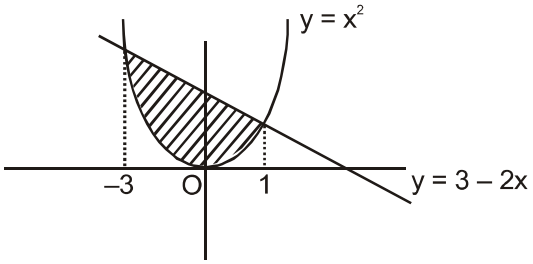
$= 106 - (10.1)^2$

$= 3.99$

7. Find area bounded by $x^2 \le y \le 3 - 2x$ $\{x, y \in R^2\}$

Ans. $\frac{32}{3}$

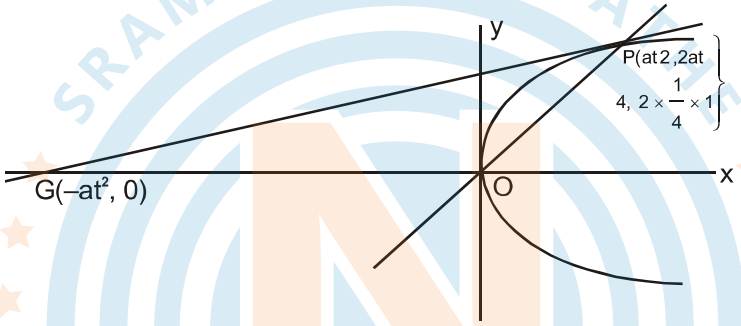
Sol. $y = 3 - 2x$



$$\text{Area} = \int_{-3}^{1} \left((3-2x) - x^2\right) dx$$

$$= \frac{32}{3}$$

8. If curves $y^2 = x$ & $y = mx$ intersect at P other than orgin, also tangent at P intersects x-axis at Q. Such that $ar(\Delta POQ) = 4$. Find m.



$ty = x + at^2 \Rightarrow Q(-at^2, 0)$

$\therefore 4 = \frac{1}{2}.at^2.2at$

$\Rightarrow t^3 = 4^3$

$t = 4$

$\therefore m = \frac{2}{t} = \frac{1}{2}$

9. In expansion of $(x+\sqrt{x^2-1})^6 + \left(x-\sqrt{x^2-1}\right)^6$ coefficient of x^4 is α & coefficient of x^2 is β then

(1) $\alpha + \beta = 36$ (2) $\alpha + \beta = 66$ (3) $\alpha - \beta = -132$ (4) $\alpha - \beta =$

Ans. (3)

Sol. Let $\sqrt{x^2-1} = k$

$(x + k)^6 + (x - k)^6$

after expansion

$2[({}^6C_0 + {}^6C_2 + {}^6C_4 + {}^6C_6)\, x^6 + x^4(-{}^6C_2 - 2 \times {}^6C_4 - 3 \times {}^6C_6) + x^2({}^6C_4 + 3 \times {}^6C_6)]$

$\alpha = 2(-{}^6C_2 - 2 \times {}^6C_4 - 3)$ $\beta = 2 \times {}^6C_4 + 2 \times 3 \times {}^6C_6$

$\alpha = -96$ $\beta = 36$

$\alpha - \beta = -132$

10. If $a = (1+\omega)\sum_{k=0}^{100}\alpha^{2k}$ & $b = \sum_{k=0}^{100}\alpha^{3k}$

where $\alpha = -\frac{1}{2}+\frac{i\sqrt{3}}{2}$

then find quadratic equation having roots a & b

Sol. $\alpha = \omega = -\frac{1}{2}+\frac{i\sqrt{3}}{2}$ where $\omega^3 = 1$

$a = (1+\omega)\sum_{k=0}^{100}\alpha^{2k}$

$= (1+\omega)\sum_{k=0}^{100}\omega^{2k}$

$= (1+\omega)(1+\omega^2+\omega^4+\omega^6 ..... +\omega^{198}+\omega^{200}]$

$= (1+\omega)(1+\omega^2)$

$= (1+\omega)(-\omega)$

$a = 1$

$b = \sum_{k=0}^{100}\alpha^{3k} = \sum_{k=0}^{100}\omega^{3k} = 100$

$\therefore$ $a + b = 101$

$ab = 100$

equation is $x^2 - 100x + 101 = 0$

11. If $I = \int_1^2 \frac{1}{\sqrt{2x^3-9x^2+12x+4}}dx$ then

which of the following is correct?

(1) $\frac{1}{8} < I^2 < \frac{1}{7}$ (2) $\frac{1}{9} < I^2 < \frac{1}{8}$ (3) $\frac{1}{10} < I^2 < \frac{1}{9}$ (4) $\frac{1}{11} < I^2 < \frac{1}{10}$

Ans. (2)

Sol. $f(x) = 2x^3 - 9x^2 + 12x + 4$

$f'(x) = 6x^2 - 18x + 12$

$= 6(x-1)(x-2)$

So in (1, 2) f(x) is decreasing

$\Rightarrow$ $\frac{1}{\sqrt{f(x)}}$ is increasing

$\frac{1}{\sqrt{f(1)}} < \int_1^2 \frac{1}{\sqrt{f(x)}}dx < \frac{1}{\sqrt{f(2)}}$

$\frac{1}{\sqrt{9}} < I < \frac{1}{\sqrt{8}}$

$\frac{1}{9} < I^2 < \frac{1}{8}$

12. If (±6, 0) are the vertices of hyperbola such that it passes through (10, 16). Then equation of normal at P(10, 16) is

Sol. $a = 6 \Rightarrow \frac{x^2}{36} - \frac{y^2}{b^2} = 1$

Passes through P(10, 16)

$\frac{100}{36} - \frac{16^2}{b^2} = 1$

$b^2 = 144$

$b = 12$

$H : \frac{x^2}{36} - \frac{y^2}{144} = 1$

Normal at P(10, 16) is

$\frac{36x}{10} + \frac{144y}{16} = 36 + 44$

$\frac{18}{5}x + 9y = 180$

$2x + 5y = 100$

13. Find the value of $\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4}$

Sol. $\sum_{n=1}^{7} \frac{n(n+1)(2n+4)}{4} - \sum_{n=1}^{7} \frac{3n(n+1)}{4} = \frac{n(n+1)(n+2)(n+3)}{2\times 4} - \frac{3}{4}\frac{n(n+1)(n+2)}{3}$

Put $n = 7 \rightarrow \frac{7.8.9.10}{8} - \frac{3}{4}\frac{7.8.9}{3} = 630 - 126 = 504$

14. If 10^{th} term of on A.P. is $\frac{1}{20}$ and 20^{th} term is $\frac{1}{10}$. Find the sum of first 200 terms of A.P.

Sol. $T_{10} = \frac{1}{20} \Rightarrow a + 9d = \frac{1}{20}$

$T_{20} = \frac{1}{10} \Rightarrow a + 19d = \frac{1}{10}$

$10d = \frac{10}{200} \Rightarrow d = \frac{1}{200}$ and $a = \frac{1}{200}$

$\therefore S_{200} = \frac{200}{2}\left[2\times\frac{1}{200} + 199\times\frac{1}{200}\right] = 100\left[\frac{201}{200}\right] = 100.5$

15. Find the range of $f(x) = \frac{x[x]}{x^2+1}$, $x \in (1, 3)$

Sol. For $1 < x < 2$

$f(x) = \frac{x}{x^2+1}$

$f(x) \in \left(\frac{2}{5}, \frac{1}{2}\right)$

for $2 \le x < 3$

$f(x) = \frac{2x}{x^2+1}$

$f(x) \in \left(\frac{3}{5}, \frac{4}{5}\right]$

$f(x) \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

16. $\lim_{x\to 0} \frac{\int_0^x t\sin(10t)dt}{x}$ equals .

(1) $-\frac{1}{10}$ (2) $\frac{1}{10}$ (3) 0 (4) $\frac{1}{100}$

Sol. $\lim_{x\to 0} \frac{\int_0^x t\sin(10t)dt}{x} = \lim_{x\to 0} \frac{x\sin(10x)}{1} = 0$

17. If $y = mx + C$ is tangent to $(x-3)^2 + y^2 = 1$ and perpendicular to the tangent to the $x^2 + y^2 = 1$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

then which option is correct .

(1) $c^2 + 6c + 7 = 0$ (2) $c^2 - 6c + 7 = 0$ (3) $c^2 + 6c - 7 = 0$ (4) $c^2 - 6c - 7 = 0$

Sol. Tangent at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ at $x^2 + y^2 = 1$

is $\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 1$

slope $= -1 \Rightarrow m = 1$

$y = x + c$ is tangent to $(x-3)^2 + y^2 = 1$

$\Rightarrow \left|\frac{3 + c - 0}{\sqrt{2}}\right| = 1$

$C = -3 \pm \sqrt{2}$

18. Which of the following is a tautology

(1) $\sim(p \vee \sim q) \rightarrow (p \wedge q)$ (2) $\sim(p \wedge \sim q) \rightarrow (p \vee q)$ (3) $\sim(\sim p \vee q) \rightarrow (p \vee q)$ (4) None

Ans. (2)

Sol.

p	q	$\sim q$	$p \vee \sim q$	$\sim (p \vee \sim q)$	$p \wedge q$	$\sim (p \vee \sim q) \rightarrow (p \wedge q)$	$p \vee q$	$\sim (p \vee \sim q) \rightarrow (p \vee q)$
T	T	F	T	F	T	T	T	T
T	F	T	T	F	F	T	T	T
F	T	F	F	T	F	F	T	T
F	F	T	T	F	F	T	F	T

is tautology